To trisect an angle, for example, the angle AOC, by means of this curve, produce CO to E and draw EA. Also draw OH; then FO drawn parallel to EA makes the angle $FOC=\frac{1}{3}\angle AOC$. For since $EH=HO$, by construction of the curve, $\angle OEH=\angle EOH$. But $\angle OHA,=\angle OAH,=\angle OEH+\angle EOH=2\angle OEH$. Hence, $\angle OEH+\angle OAE,=3\angle OEH,=\angle AOC$, or $\angle OEH,=\angle FOC=\frac{1}{3}\angle AOC$.

After this department, in the last issue, was in type, we received solutions of problem 299 from Professors Scheffer, Zerr, and Greenwood. Professors Scheffer and Greenwood's solutions consisted in connecting a point, G, of the ellipse with the foci F, F'. M, the middle point of FG, is taken for the center of the circle described on the focal radius, FG, as a diameter. The line AM joining M and A, the center of the ellipse, is $\frac{1}{2}F'G$, since $AF=AF'$ and M is the middle point of FG. But $\frac{1}{2}AF'=\frac{1}{2}(2a-AF)=a-\frac{1}{2}AF$, from the definition of the ellipse. Hence, MA, the distance between the centers of the auxiliary circle and the circle described on $AF=a-\frac{1}{2}AF$, the difference of their radii. Hence the circles touch.

Dr. Zerr's solution, which was analytical, made use of the same property.

CALCULUS.

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228. Proposed by B. F. FINKEL, Ph. D., Professor of Mathematics and Physics, Drury College, Springfield, Mo.

A sphere, radius r, is dropped into a conical vessel whose vertex angle is 60°. Find the contents of the vessel between the vertex and the sphere by means of the formula, $V=\int\int\int dx\,dy\,dz$.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va., and the PROPOSER.

$x^2+y^2+z^2=r^2$ is the equation to the sphere, and $x^2+y^2=\frac{1}{3}(2r-z)^2$ is the equation to the cone. Eliminating z we get $y=\sqrt{(\frac{3}{4}r^2-x^2)}$.

$\therefore y=\sqrt{(\frac{3}{4}r^2-x^2)}=y'$ to $y=0$, $x=\frac{1}{2}r\sqrt{3}=x'$ to $x=0$.

$$\therefore\ v=4\int_0^{x'}\int_0^{y'}[2r-\sqrt{3}\sqrt{(x^2+y^2)}-\sqrt{(r^2-x^2-y^2)}]dxdy$$

$$=4\int_0^{x'}\Bigg[r\sqrt{(\tfrac{3}{4}r^2-x^2)}-\tfrac{1}{2}(r^2-x^2)\sin^{-1}\sqrt{\frac{\frac{3}{4}r^2-x^2}{r^2-x^2}}$$

$$-\tfrac{1}{2}\sqrt{3}\,x^2\log\left(\frac{\sqrt{(\frac{3}{4}r^2-x^2)}+\frac{1}{2}r\sqrt{3}}{x}\right)\Bigg]dx$$

$$=4(\tfrac{3}{16}\pi r^3-\tfrac{1}{64}\pi r^3+\tfrac{1}{12}\pi r^3-\tfrac{1}{6}\pi r^3-\tfrac{3}{64}\pi r^3)=\tfrac{1}{6}\pi r^3.$$

229. Proposed by J. EDWARD SANDERS, Reinersville, Ohio.

Solve the differential equation $d^2y/dx^2=axy$.

Solution by S. A. COREY, Hiteman, Iowa, and LEROY D. WELD, Coe College, Cedar Rapids, Iowa.

Let $y=c_0+c_1x+c_2x^2+c_3x^3+c_4x^4+c_5x^5+$ etc. ...(1).